

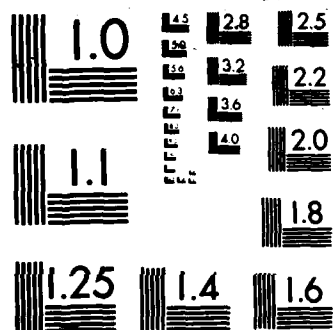
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**NONLINEAR ACOUSTICS: LONG RANGE UNDERWATER PROPAGATION,  
NONCOLLINEAR INTERACTION, REFLECTION AND REFRACTION,  
AND SUBHARMONIC GENERATION  
SECOND ANNUAL SUMMARY REPORT  
UNDER CONTRACT N00014-84-K-0574**

David T. Blackstock

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5 December 1986

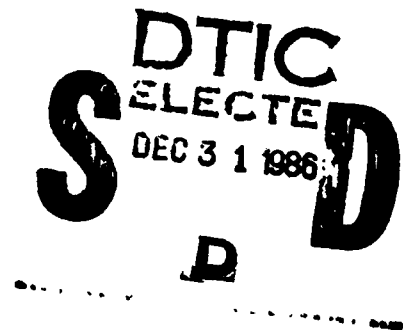
Technical Report

1 November 1985 - 31 October 1986

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*Prepared for:*

**OFFICE OF NAVAL RESEARCH  
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Research on four topics in nonlinear acoustics is reported. (1) Nonlinear effects in underwater propagation. Two analytical-computational projects in this area are discussed: (a) long range propagation of finite-amplitude pulses in a stratified, dissipative ocean, and (b) the pressure, temperature, and salinity dependence of the nonlinearity coefficient $\beta$ and related quantities. (2) Nonlinear, noncollinear interaction of sound waves.		

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Two analytical and experimental studies of interaction in a rectangular waveguide are described. First, a monochromatic wave is generated in the 1,0 mode of the waveguide, and distortion produces a second harmonic component in the 2,0 mode, a third harmonic component in the 3,0 mode, etc. and so on. Second, two primary waves are generated, a low frequency wave in the 0,0 mode and a high frequency wave in the 1,0 mode. Of main interest are the sum and difference frequency waves produced by the nonlinear interaction of the two primaries. In a third study, which is not limited to waveguides, the angular dependence of the nonlinearity coefficient  $\beta$  was investigated. (3) Reflection and refraction of finite-amplitude sound at a plane interface between two fluids. The purpose of this work, which has just begun, is to determine the applicability of Snell's law, the law of specular reflection, and other familiar small-signal results when the incident wave is of finite amplitude. A literature survey is reported. (4) Investigation of subharmonic generation and chaos in an acoustical resonance tube. On the basis of a careful survey of the literature, it is concluded that a closed end resonance tube is not likely to exhibit significant subharmonic response. Chaotic behavior is therefore not expected.

Four technical reports are listed along with five oral papers, four papers in proceedings, and four articles submitted for journal publication.

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## TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION	1
II. PROJECTS	1
A. Nonlinear Effects in Underwater Propagation	1
B. Nonlinear, Noncollinear Interaction of Sound Waves	6
C. Reflection and Refraction of Finite-Amplitude Sound at a Plane Interface Between Two Fluids	9
D. Investigation of Subharmonic Generation and Chaos in an Acoustical Resonance Tube	13
E. Other Work	16
APPENDIX A     A STUDY OF POSSIBLE SUBHARMONIC GENERATION IN A CLOSED END RESONANCE TUBE	17
CHRONOLOGICAL BIBLIOGRAPHY, 1984-1986	29



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## I. INTRODUCTION

The research carried out under Contract N00014-84-K-0574, which began 1 August 1984, is primarily in the field of nonlinear acoustics. The broad goal is to determine the laws of behavior of finite-amplitude sound waves, especially to find departures from the laws of linear acoustics. Phenomena are studied that may have application to problems of national defense and technology. The contract is the successor to Contract N00014-75-C-0867, which ended 31 August 1984 (86-1),<sup>1</sup> and also continues research begun under Contract N00014-82-K-0805 on the specific topic of nonlinear effects in long range underwater propagation (86-3). This report covers the period 1 November 1985 - 31 October 1986 (the First Annual Report (85-8) covered the period 1 August 1984 - 31 October 1985). However, much of the work reported here had its beginnings under the two previous contracts.

The following persons participated in the research.

### Graduate students

F. D. Cotaras, Ph.D. student in Electrical and Computer Engineering.

Andrew J. Kimbrough, M.S. student in Mechanical Engineering.

J. A. TenCate, Ph.D. student in Mechanical Engineering.

### Senior personnel

C. L. Morfey, consultant, Institute of Sound and Vibration Research, University of Southampton, England.

W. M. Wright, consultant, Physics Department, Kalamazoo, Michigan.

D. T. Blackstock, principal investigator.

## II. PROJECTS

### A. Nonlinear Effects in Underwater Propagation

The main project in this area has been long range propagation of finite-amplitude pulses in a stratified, dissipative ocean. Additional special projects include (a) the

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<sup>1</sup> Numbers given in this style refer to items in the Chronological Bibliography given at the end of this report, e.g., 86-1 means the first entry in the list for 1986.

pressure, temperature, and salinity dependence of the nonlinearity coefficient  $\beta$  and related quantities, and (b) passage of a finite-amplitude pulse through a caustic region. For descriptions of work done prior to the period of this report, see Refs. 85-8 and 86-3.

1. Finite-Amplitude Pulse Propagation through a Layered Dissipative Ocean

The primary purpose of this project, which was carried out mainly by Cotaras, was to determine the extent to which nonlinear propagation effects are important in long range underwater propagation. Nonlinear geometrical acoustics (ray theory for finite-amplitude waves) was the theoretical model used, and the approach was both analytical and computational. The acoustical signals studied were largely limited to explosion pulses. Taken into account in the propagation were the inhomogeneity (layered structure) of the ocean medium, finite-amplitude distortion, and real-ocean attenuation and dispersion.

With the completion of Cotaras's master's thesis in August 1985 (85-7), the work was largely completed. (Not considered, however, were propagation paths that include reflections or caustics.) A substantial amount of time during the present year was devoted to reporting the results in the open literature. First, Cotaras converted his thesis to a technical report (85-7). Then two oral and written papers were prepared for the 12th International Congress on Acoustics and one of its satellite symposia, Underwater Acoustics, which were held in Canada in July 1986. The abstracts of the two papers (86-4 and 86-5, respectively) are as follows:

Beyond What Distance are Finite Amplitude Effects Unimportant?

F. D. Cotaras, D. T. Blackstock, and C. L. Morfey, 12th ICA Associated Symposium on Underwater Acoustics, Halifax, Canada, 16-18 July 1986, Paper Q-9.

The propagation of explosion pulses through a lossy, stratified ocean was investigated numerically via nonlinear geometrical acoustics. Reflections and focusing were not considered. Calculations for a weak shock with an exponentially decaying tail were made to determine the relative importance of nonlinear distortion, inhomogeneity, losses, and dispersion on the received



wave. More realistic waveforms (one bubble pulse included) were used to determine the distance beyond which nonlinear effects are unimportant. For a 0.818 kg TNT explosion, the distance was found to be 1100 m for spectral components below 6 kHz. For the 22.7 kg explosion, the corresponding quantities are 4 kHz and 1100 m.

#### Time Domain Presentation of Geometrical Acoustics

Frederick D. Cotaras and David T. Blackstock, 12th International Congress on Acoustics, Toronto, Canada, 21-31 July 1986, Paper I2-1.

The theories of linear and nonlinear geometrical acoustics are derived from simplified versions of the lossless hydrodynamics equations in the fashion put forth by Ostrovskii et al. (Sov. Phys.-Acoust. 22, 516-520, 1976). The development, which is carried out solely in the time domain, leads to an eikonal equation and a transport equation. The eikonal equation, from which an equation for the ray paths is derived, is the same for both small-signal and finite amplitude waves. The transport equation is, however, different for the two cases. It leads to a standard first-order progressive wave equation, linear for small-signal waves, but nonlinear for finite amplitude waves.

## 2. Special Projects

Discussed in this section are two special projects. First is calculation of the dependence of various coefficients of nonlinearity on pressure, temperature, and salinity (Morfeý, Cotaras, and Kimbrough). This work is an outgrowth of calculations done earlier by Morfeý (under Contract N00014-82-K-0805) during the early stages of the long range propagation project.<sup>2</sup>

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<sup>2</sup> C. L. Morfeý, "Nonlinear Propagation in a Depth-Dependent Ocean," Technical Report ARL-TR-84-11, Applied Research Laboratories, The University of Texas at Austin, 1 May 1984 (ADA 145 079).

The best known coefficient of nonlinearity is  $\beta$ ,

$$\beta = 1 + B/2A = 1 + \rho_0 c_0 (\partial c / \partial P)_T + (\alpha c_0 T_0 / C_p) (\partial c / \partial T)_P, \quad (1)$$

where  $\rho$  is density,  $c$  is sound speed,  $P$  is total pressure,  $T$  is absolute temperature,  $\alpha$  is the isobaric coefficient of thermal expansion,  $C_p$  is the specific heat at constant pressure, and  $B/2A$  is the dimensionless coefficient of the quadratic term in the isentropic pressure-density relation.<sup>3</sup> The subscript 0 denotes a value for the undisturbed medium. It is also understood that the partial derivatives are evaluated at  $\rho = \rho_0$  (the undisturbed state). In first-order nonlinear acoustics  $\beta$  is the only coefficient of nonlinearity required. For example,  $\beta$  appears in the plane wave formula for shock propagation speed

$$\left( \frac{dx}{dt} \right)_{\text{shock}} = c + \beta \frac{p_m}{\rho_0 c_0}, \quad (2)$$

where  $p_m$  is the mean of the acoustic pressure ( $P - P_0$ ) on both sides of the shock.

For strong enough waves, however, the propagation cannot be characterized by  $\beta$  alone. In second-order nonlinear acoustics a new coefficient  $\beta'$ , given by

$$\beta' = \rho c^2 (\partial \beta / \partial P)_S = \beta (\beta - 1) + \rho^2 c^3 (\partial^2 c / \partial P^2)_S \quad (3)$$

( $S$  stands for entropy), must be introduced. The new coefficient appears, for example, in the second-order version of Eq. (2),

$$\left( \frac{dx}{dt} \right)_{\text{shock}} = c + \beta \frac{p_m}{\rho_0 c_0} + \frac{c_0}{6} \left( \frac{\Delta P}{\rho_0 c_0^2} \right) \left( \frac{\phi \beta}{2} - \beta (\beta - 1) + \frac{\beta'}{4} \right). \quad (4)$$

<sup>3</sup> H. Endo, "Determination of the nonlinearity parameters for liquids using thermodynamic constants," J. Acoust. Soc. Am. 71, 330-333 (1982).

Here  $\Delta P$  is the pressure jump across the shock, and

$$\phi = \alpha c^2 / C_p \quad (5)$$

is the Gruneisen parameter. An alternative expression for  $\beta'$  is

$$\beta' = C/2A - (\beta-1)(\beta-3)/2 \quad , \quad (6)$$

where  $C/2A$  is the dimensionless coefficient of the cubic term in the isentropic pressure-density relation.

Using data on the properties of  $c$ ,  $\rho$ ,  $\alpha$ , and  $C_p$  for seawater, we expect to calculate values of  $\beta$ ,  $\beta'$ ,  $\phi$ , and related quantities as a function of pressure, temperature, and salinity. Calculations of this sort for  $\beta$  have recently been reported by Endo.<sup>3,4</sup> Endo's results will serve as a check for our computations. Our calculations will be based on data from sources<sup>5-9</sup> in addition to those Endo used.<sup>10-12</sup> During the present report period we have compiled on a computer the various data bases needed to calculate  $\beta$ ,  $\beta'$ ,  $\phi$ , and related quantities.

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<sup>4</sup>H. Endo, "Calculation of nonlinearity parameter for seawater," J. Acoust. Soc. Am. 76, 274-279 (1984).

<sup>5</sup>B. Gebhart and J. C. Mollendorf, "A new density relation for pure and saline water," Deep Sea Res. 24, 831-848 (1977).

<sup>6</sup>J. R. Lovett, "Merged seawater sound-speed equations," J. Acoust. Soc. Am. 63, 1713-1718 (1978).

<sup>7</sup>C-T. Chen, R. A. Fine, and F. J. Millero, "The equation of state of pure water determined from sound speeds," Chem. Phys. 66, 2142-2144 (1977).

<sup>8</sup>Engineering Sciences Data Unit, ESDU Data Item No. 77024 (London).

<sup>9</sup>Engineering Sciences Data Unit, ESDU Data Item No. 68008 (London).

<sup>10</sup>C-T. Chen and F. J. Millero, "The specific volume of seawater at high pressures," Deep Sea Res. 23, 595-612 (1976).

<sup>11</sup>C-T. Chen and F. J. Millero, "Speed of sound in seawater at high pressures," J. Acoust. Soc. Am. 62, 1129-1135 (1977).

<sup>12</sup>F. J. Millero, G. Perron, and J. E. Desnoyers, "Heat capacity of seawater solutions from 5° to 35°C and 0.5% to 22% chlorinity," J. Geophys. Res. 78, 4499-4507 (1973).

The second special project is propagation of finite-amplitude pulses through caustics. Work on this topic was done during the previous year by Richard Buckley, a consultant at University of Southampton, under the direction of Morfey. See Ref. 85-8. Unfortunately, because of the illness of Buckley during the present year, no progress may be reported.

B. Nonlinear, Noncollinear Interaction of Sound Waves

Work has been done on three tasks in this area: (1) Case A interaction in a rectangular waveguide, (2) Case B interaction in a rectangular waveguide, and (3) angular dependence of  $\beta$  for two interacting wave fields. Cases A and B are defined below. The work has been carried out chiefly by TenCate, in collaboration with Mark F. Hamilton (whose support since 1 September 1985 has come from Contract N00014-85-K-0798<sup>13</sup>) and, to a lesser extent, Blackstock.

In our previous annual report (85-8) two different cases of higher mode propagation of finite-amplitude waves in a rectangular waveguide are described. In Case A (called the "second case" by Hamilton<sup>13</sup>) a single source launches an intense monochromatic wave (the fundamental) that travels down the waveguide in the 1,0 mode. Nonlinear propagation effects produce a family of distortion components to accompany the fundamental: the second harmonic in the 2,0 mode, the third harmonic in the 3,0 mode, the fourth harmonic in the 4,0 mode, and so on. The analysis is simplest and the experiments most easily explained when the fundamental is in the intermediate range neither very close to cutoff nor very far from cutoff. Case B (called the "first case" by Hamilton<sup>13</sup>) involves the noncollinear interaction of a low frequency wave in the 0,0 mode with a high frequency wave in the 1,0 mode. To be consistent with limitations imposed by the analysis, TenCate and Hamilton carried out experiments only with relatively weak waves.

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<sup>13</sup> M. F. Hamilton, "Problems in Nonlinear Acoustics: Parametric Receiving Arrays, Focused Finite Amplitude Sound, and Dispersive Nonlinear Interactions," First Annual Report under Contract N00014-85-K-0708, Department of Mechanical Engineering, The University of Texas at Austin, April 1986, Section III.

A preliminary report of our work on Cases A and B was given during the first year of the contract (85-8). During the summer of 1985 experimental work on Case B showed frustrating inconsistencies; it seemed very difficult to reproduce results from one day to the next. It was finally discovered that the problems were due to spatial aliasing and to the important role played by local nonlinear effects. We are indebted to the Tjøttas<sup>14</sup> for instruction about the latter.

A second oral paper was given at the Nashville Meeting of the Acoustical Society of America in November 1985 (85-9). Stressed in this paper were the theory for Case B and analysis relating to material on the third task, i.e., the angular dependence of  $\beta$ . The abstract of the paper is as follows:

*A coefficient of nonlinearity for noncollinear plane wave interaction*

Mark F. Hamilton and James A. TenCate, 110th Meeting, Acoustical Society of America, Nashville, 4-8 November 1985, Paper S1.

An inhomogeneous wave equation, exact to second order in the field variables, is derived for the sum and difference frequency pressure generated by two plane waves, of angular frequencies  $\omega_1$  and  $\omega_2$ , which intersect at an angle  $\phi$  in a lossless fluid. The coefficient of nonlinearity pertaining to the sum or difference frequency wave ( $\omega_{\pm} = \omega_1 \pm \omega_2$ ) is shown to be

$$\beta_{\pm}(\phi) = B/2A + \cos\phi \pm 4(\omega_1\omega_2/\omega_{\pm}^2) \sin^4(\phi/2) , \quad (7)$$

where  $B/A$  is the parameter of nonlinearity. The same result may be deduced from the work of Zverev and Kalachev [Sov. Phys.-Acoust. 15, 322 (1970)]. The first term of  $\beta_{\pm}$  is due to the isentropic equation of state, the second term represents convection, and the third term comes from the momentum equation.

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<sup>14</sup> J. N. Tjøtta and S. Tjøtta, "Interaction of sound waves, Part I: Basic equations," submitted to J. Acoust. Soc. Am.

Alternative formulations of the inhomogeneous wave equation are presented, and comparisons are made with the analyses of others. An experiment designed to measure the angular dependence of  $\beta_{\pm}$  was conducted with noncollinear waves in an airfilled waveguide. Results are reported.

A third oral paper, accompanied by an extended written summary, was presented at the 12th International Congress on Acoustics at Toronto in July 1986 (86-6). More details and new measurements were reported. Both Case A and Case B were covered, but only a little was said about the discovery of the importance of local nonlinear effects in the Case B interaction. A full discussion of these effects appears in a written paper submitted for publication in the Journal of the Acoustical Society of America in September 1986 (86-9). The abstract of this paper is as follows:

Sum and Difference Frequency Generation Due to Noncollinear Wave Interaction in a Rectangular Duct

Mark F. Hamilton and James A. TenCate

Noncollinear wave interaction in a rectangular duct is investigated both theoretically and experimentally. An inhomogeneous wave equation, exact to second order in the field variables, is derived for the sum and difference frequency pressure waves generated by noncollinear interaction of two finite amplitude plane waves in a lossless fluid. This equation is extended to the interaction of waves in higher order modes of a rectangular duct. Quasilinear solutions are obtained, and tube wall attenuation is included ad hoc. Experimental results are reported for the interaction of waves in the (0,0) and (1,0) modes of an air filled rectangular duct. Theory and experiment are in excellent agreement with regard to oscillatory structure of the sum and difference frequency wave fields. Although agreement between theory and experiment is reasonable ( $\pm 2$  dB), it is not within estimated experimental error ( $\pm 1$  dB). It is shown that because local rather

than cumulative nonlinear effects dominate the interaction, knowledge of the proper second order source condition is of crucial importance. Discrepancies between the predicted and measured amplitudes are attributed to an inadequate description of the source condition.

Two more written papers are planned. One, on the angular dependence of  $\beta$  (86-10), will be submitted to J. Acoust. Soc. Am. in 1986. The other, on Case A interaction, will be submitted by TenCate and Hamilton in 1987.

### C. Reflection and Refraction of Finite-Amplitude Sound at a Plane Interface Between Two Fluids

This project is a prime example of our attempt to discover the laws of nonlinear acoustics. The harvest of postwar research in nonlinear acoustics, by investigators in many lands, is impressive. We now know a great deal about the propagation and absorption of finite-amplitude waves, including plane, spherical, and cylindrical waves and even quasi-one-dimensional waves, such as waves traveling through horns or ray tubes. Research on parametric arrays has led to important discoveries about piston-type radiation (a difficult two-dimensional problem) and diffraction of finite-amplitude waves. Left relatively untouched in the charge of progress, however, have been the important phenomena of reflection and refraction.

#### 1. Background

Two of the oldest and most venerable laws of acoustics (and many other fields of wave motion) are the law of specular reflection and Snell's law. These laws give the angles of reflection  $\phi_r$  and transmission  $\phi_t$ , respectively, when a plane wave is incident at an angle  $\phi_i$  on a plane interface between two fluids, in particular,

$$\text{Specular reflection: } \phi_r = \phi_i, \quad (8)$$

$$\text{Snell's law: } \frac{\sin \phi_t}{\sin \phi_i} = \frac{c_2}{c_1}, \quad (9)$$

where  $c_1$  and  $c_2$  are the sound speeds in the first and second fluids, respectively.

Another important result has to do with the pressures of the incident ( $p_i$ ), reflected ( $p_r$ ), and transmitted ( $p_t$ ) waves. For example, at a rigid surface we have the following result:

$$\text{Rigid surface:} \quad p_{\text{surface}} = 2 p_i , \quad (10)$$

where  $p_{\text{surface}}$  is the combined pressure of the reflected and incident waves at the surface. This law is known as pressure doubling.

But do these laws hold for finite-amplitude sound waves? All known derivations are based on linear theory, that is, small-signal waves are assumed. Two possibilities for investigation are clear. Either (1) the laws continue to hold when the amplitude becomes finite, in which case it should be possible to generalize their derivation, or (2) the laws change with amplitude, in which case the nature and extent of the change needs to be determined.

Not much evidence is available on reflection and refraction of finite-amplitude waves, but that which does exist tends to support the second possibility. First, it is known that when a finite-amplitude sound wave strikes a rigid wall at normal incidence, deviations from pressure doubling do occur.<sup>15</sup> The rigorously correct law is that the excess sound speed ( $c-c_0$ ) doubles. Since the acoustic pressure  $p$  is nonlinearly related to the excess sound speed, pressure doubling occurs only in the small-signal limit, where the relation between  $p$  and  $c-c_0$  becomes linear. Although the case of normal incidence does not bear directly on the question of reflection and refraction at oblique incidence, the fact that one known reflection law fails to hold at high amplitude implies that others may fail as well. Second, and of more direct relevance, is the information known about reflection and refraction of steady shock

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<sup>15</sup> D. T. Blackstock, "Normal reflection of finite amplitude plane waves from a rigid wall," in Proc. of the Third Intern. Cong. Acoust., Stuttgart, 1959, edited by L. Cremer (Elsevier Pub. Co., Amsterdam, 1961), Vol. I, pp. 309-311.



waves obliquely incident on a plane interface between two gases.<sup>16,17</sup> Experiments show that while specular reflection and Snell's law hold for weak shocks, they do not hold for stronger shocks. Moreover, phenomena foreign to small-signal acoustical theory are observed. The interface itself undergoes a small but finite angular deflection. More interesting is the fact that beyond a certain shock strength the incident, reflected, and transmitted rays no longer intersect at the interface. The incident and reflected rays do intersect but at a point above the interface; their intersection is joined to the interface by a new shock, called the stem shock. Unfortunately, the results for steady shock waves, which are both theoretical and experimental, cannot be applied directly, or even easily modified, to continuous sound waves of finite amplitude. Steady shocks are very special waves, having constant states before and aft, and this fact greatly simplifies the analysis of their behavior.

In a few instances the question of reflection and refraction appears in the literature of nonlinear acoustics. Van Buren and Breazeale<sup>18,19</sup> simply assume that reflection and refraction are linear processes. For example, given a finite-amplitude wave incident on a reflecting boundary, the authors decompose the (distorted) wave into its *Fourier components*, each of which is then treated independently (and linearly) to calculate its reflection. The reflected signals are then recombined and the composite reflected wave is then allowed to propagate backward, distorting as it goes. This approximation is very straightforward and useful (a number of other investigators have used it), but it sheds no light on the fundamental question of the validity of specular reflection and Snell's law. In nonlinear geometrical acoustics the question is also dodged by simply assuming that the ray paths are not changed when the signal is of finite amplitude (85-7).

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<sup>16</sup> R. G. Jahn, "The refraction of shock waves at a gaseous interface," J. Fluid Mech. 1, 457-489 (1956).

<sup>17</sup> R. G. Jahn, "Transition processes in shock wave interactions," J. Fluid Mech. 2, 33-48 (1957).

<sup>18</sup> A. L. Van Buren and M. A. Breazeale, "Reflection of finite-amplitude ultrasonic waves. I. Phase shift," J. Acoust. Soc. Am. 44, 1014-1020 (1968).

<sup>19</sup> A. L. Van Buren and M. A. Breazeale, "Reflection of finite-amplitude ultrasonic waves. II. Propagation," J. Acoust. Soc. Am. 44, 1021-1027 (1968).

## 2. Progress during the Current Report Period

The investigation of reflection and refraction is Cotaras's doctoral research topic. He has not been able to devote much time to it during the current year, however, because of the effort required to finish up the long range propagation project (85-7, 86-4, 86-5).

A literature survey has been done. Besides the works already referred to,<sup>15-19</sup> papers by Ginsberg<sup>20-22</sup> and two Chinese investigators<sup>23-26</sup> have been reviewed. Ginsberg's papers are about finite-amplitude sound generated in a fluid by transverse surface waves on a plate bounding the fluid. It was first thought that Ginsberg's results might be directly applicable to the reflection-refraction problem. In both cases the radiated waves (one field in Ginsberg's problem, two fields - reflected and transmitted - in the reflection-refraction problem) are generated by a traveling surface disturbance. In Ginsberg's case the surface is the vibrating plate. In the reflection-refraction problem the surface is the interface, which is set in motion by the incident wave. Unfortunately, however, it seems the two problems really are different. The plate vibration is regarded as given, for example, a pure sinusoidal function, whereas the the interface motion is a response coupled to the incident, reflected, and refracted fields. As a rough approximation one would expect the interface wave simply

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<sup>20</sup> J. H. Ginsberg, "Multi-dimensional non-linear acoustic wave propagation, Part II: The nonlinear interaction of an acoustic fluid and plate under harmonic excitation," J. Sound Vib. 40, 359-379 (1975).

<sup>21</sup> J. H. Ginsberg, "A re-examination of the non-linear interaction between an acoustic field and a flat plate undergoing harmonic excitation," J. Sound Vib. 60, 449-458 (1978).

<sup>22</sup> J. H. Ginsberg "Nonlinear generation of harmonics in sound radiation from a vibrating planar boundary," J. Acoust. Soc. Am. 69, 60-65 (1981).

<sup>23</sup> Z. Qian, "Reflection of finite-amplitude sound wave on a plane boundary of half space," Scientia Sinica (Series A) 25, 492-501 (1982).

<sup>24</sup> Z. W. Qian, "Reflection of finite-amplitude sound wave on a plane boundary of half space (II)," Fortschritte der Akustik FASE/DAGA '82, Gottingen, 821-824 (1982).

<sup>25</sup> S. Feng, "Reflection of finite amplitude waves," Sov. Phys.-Acoust. 6, 488-490 (1961).

<sup>26</sup> S. Feng, "The reflection and refraction of a large amplitude plane sound wave in two dimensions," Chinese J. Acoust. 2, 291-302 (1983).

to track the incident wave as in linear acoustics. If this is the case, the interface wave must distort as it travels, not remain a pure sinusoid.

The papers by Qian<sup>23,24</sup> and Feng<sup>25,26</sup> (and related papers cited by them) are the only ones known to the author that deal specifically with nonlinear effects in oblique-incidence reflection and refraction. The approach is to solve the two-dimensional (lossless) equations of motion by conventional second order methods, such as ordinary perturbation. Since the incident wave is assumed to be (initially) monochromatic, the second order solution is composed of second harmonic signals.<sup>23-25</sup> For example, for this case Qian<sup>23</sup> finds three second harmonic waves: one associated with progressive distortion of the incident wave, another associated with progressive distortion of the reflected wave (he claims that no causal relation exists between the two), and finally an entirely new wave, which he calls a "Q-harmonic," that propagates parallel to the rigid surface. The Q wave results from nonlinear interaction of the primary incident and reflected waves. Feng<sup>26</sup> obtains a full second order solution for the two-fluid problem. Second order as well as first order reflection and transmission coefficients are identified. It is interesting to note that the second order coefficients depend on the values of  $\beta$  for the two fluids. Other interesting results include the apparent variation of amplitude along the wavefronts of the second harmonic reflected and transmitted waves.

An oblique incidence experiment is being planned for the next period of the contract.

#### D. Investigation of Subharmonic Generation and Chaos in an Acoustical Resonance Tube

##### 1. Background

One of the most interesting fundamental questions in physics and engineering today is how a deterministic system excited by a nonstochastic driving force can develop random response. How does chaos spring unexpectedly out of order and regularity? A broad interest has developed in this question; see, for

example, the proceedings of several recent interdisciplinary conferences.<sup>27-30</sup> Chaotic response has been observed in fluid flows, chemical reactions, mechanical systems (for example, a mass on a nonlinear spring), population dynamics, nonlinear electrical circuits, and structures operating under heavy ac or dc loads. In all cases nonlinearity is an essential ingredient.

The descent to chaos frequently begins with period doubling bifurcations, that is, subharmonics appear in the response of a system. For example, a deterministic system driven just past its linear range may show only the fundamental and higher harmonic frequencies  $f$ ,  $2f$ ,  $3f$ ,  $4f$ , ..., in its output. Driving the system harder may produce subharmonics, for example, signals at  $f/2$  and  $f/4$ . As the driving force is further increased, more bifurcations occur. Signals at  $f/8$ ,  $f/16$ ,  $f/32$ , ..., appear and also some of their harmonics,  $3f/8$ ,  $5f/8$ ,  $3f/4$ ,  $3f/2$ , .... Eventually the spectrum of the output becomes so cluttered that it cannot be distinguished from noise. Chaos has set in.

Very little work has been done on chaos in acoustics. Yet many acoustical systems are basically nonlinear and operate in modes that would seem to allow the possibility of bifurcation. The only published work so far on chaos in acoustical systems is that by Rudnick and coworkers on surface waves and liquid helium,<sup>31-33</sup> and by Lauterborn and coworkers on acoustic cavitation.<sup>34-36</sup> The latter measured the sound from an acoustically generated bubble field in water and observed period doubling and eventually chaos. Although the results are unmistakable, Lauterborn's system is very complicated (the measured sound comes from a bubble field, not a single bubble), and it is not very controllable. A simpler acoustical system with which to demonstrate chaos would be desirable. Another need

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<sup>27</sup> "Testing Nonlinear Dynamics," NATO Advanced Research Workshop, Physics 11D, 252 (1984).

<sup>28</sup> "Order in Chaos," Proceedings of conference at Los Alamos, 24-28 May 1982, Physica 7D Nos. 1-3 (1983).

<sup>29</sup> "Bifurcation Theory and Applications in Scientific Disciplines," ed. O. Gurel and O. E. Roessler, Ann. N.Y. Acad. Sci. 316 (1979).

<sup>30</sup> "Nonlinear Dynamics," ed. R. Hellerman, Ann. N.Y. Acad. Sci. 357 (1980).

has to be with the kind of system to be investigated. Most previous experiments on chaos have been done with lumped element systems, such as a mass on a nonlinear spring or a nonlinear electrical circuit. Except in certain areas of hydrodynamics (and that by Rudnick and coworkers), little or no work has been done on chaotic behavior of distributed systems. A very simple, distributed acoustical system that would seem to have the potential for chaotic behavior is the closed-end resonance tube. Driven hard at one of its higher order resonances, it would have the capability of responding well to subharmonic signals should they develop.

## 2. Work during the Current Report Period

The subharmonic response of an acoustical resonance tube was to have been TenCate's doctoral project. Like Cotaras, TenCate was unable to devote much time to his project this year because of time spent on finishing up a previous task, in TenCate's case noncollinear interaction in a rectangular waveguide (85-9, 86-6, 86-9). Enough work was done, on the other hand, to show that although the resonance tube seemed attractive for demonstrating subharmonic response and chaos, in fact it does not have all the requisite characteristics.

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<sup>31</sup> R. Keolian, L. A. Turkevich, S. J. Putterman, I. Rudnick, and J. A. Rudnick, Phys. Rev. Lett. 47, 1133 (1981).

<sup>32</sup> R. Keolian and I. Rudnick, "Smooth modulation of parametrically driven surface waves in liquid helium 4," 17th Int. Conf. Low Temp. Phys., Karlsruhe, 15-22 August 1984, Proceedings, pp. 1121-1122 (North-Holland, Amsterdam, Netherlands, 1984).

<sup>33</sup> R. Keolian and I. Rudnick, "A surface wave instability on liquid helium and water," Paper 110 presented at the 109th Meeting, Acoustical Society of America, Austin, 8-12 April 1985. J. Acoust. Soc. Am. 77, S21 (1985).

<sup>34</sup> W. Lauterborn and E. Cramer, "Subharmonic route to chaos observed in acoustics," Phys. Rev. Lett. 47, 1445-1448 (1981).

<sup>35</sup> W. Lauterborn and E. Suchla, "Bifurcation superstructure in a model of acoustic turbulence," Phys. Rev. Lett. 53, 2304-2307 (1984).

<sup>36</sup> W. Lauterborn, "Acoustic turbulence," in Proceedings of the International School of Physics "Enrico Fermi", Course XCIII (North-Holland, Amsterdam, 1986), pp. 124-143.

In the beginning, we expected the resonance tube to exhibit characteristics similar to those of a mass-nonlinear spring oscillator. When driven hard, a hard spring oscillator, for example, develops a "bent over" (multivalued) response curve. A bifurcation in the motion is then possible that leads to subharmonic response and, ultimately, to chaos. Careful study of the literature on finite-amplitude standing waves in a tube showed, however, that the tube is not a good candidate for chaotic response. Although the response curve of a resonance tube does show a slight tendency to become skewed as the drive amplitude is raised, a much more pronounced effect is a lowering of the Q of the response. The broader the resonance curve, the more difficult it is for the curve to bend over so far that it becomes multivalued. Bifurcation then does not occur, and without it, significant subharmonic response is not likely.

A complete telling of the story is given in Appendix A, which was written by TenCate. Although the results of the investigation are negative, and therefore disappointing, the study was not without benefit, both educational and technical.

#### E. Other Work

Journal articles were submitted on some of our older work on other projects, in particular, thermoacoustics (86-8) and saturation in porous materials (86-2). Finally, a general review of nonlinear acoustics was presented (86-7).

## APPENDIX A

### STUDY OF POSSIBLE SUBHARMONIC GENERATION IN A CLOSED END RESONANCE TUBE

by

James A. TenCate

#### 1 INTRODUCTION

##### 1.1 Prelude

Over the past 10 years a considerable amount of attention has been focused on the chaotic behavior of nonlinear systems. Many nonlinear systems, it seems, can make a transition from periodic to chaotic behavior [1]. The most common way in which this transition occurs is the appearance of subharmonics in the response of the system. The initial idea for our study was to look for such a transition (or *route to chaos*) in a distributed acoustical system, an air-filled resonance tube. An attempt was to be made to observe subharmonics in a tube driven at one of its higher order longitudinal modes. If subharmonics *were* observed, we would try to observe the complete onset of chaos. Unfortunately, after a careful examination of related previous work, we have concluded that subharmonics would likely not be observed. What follows is a review of previous work which led to our conclusion as well as a discussion of the current state of affairs in the field of acoustic chaos. A little background is appropriate at this point.

##### 1.2 General Discussion

Two common ways for a system to make the transition to chaos are (1) *period-doubling* routes to chaos [2,3,4], and (2) *quasi-periodic* routes [5,6,7]. Many quasi-periodic routes seem to be associated with a parametric excitation of the system; period-doubling routes are often encountered with nonlinear oscillators described by a *Duffing equation* (e.g., a mass on a nonlinear spring).

The period-doubling transition to chaos is perhaps the more elegant. If one drives a nonlinear system beyond a certain critical point, the system noise (in the form of subharmonics) suddenly begins to increase rapidly. The system which initially responded at only the drive frequency now responds at  $1/2$  the forcing frequency as well. Period-doubling (or a *bifurcation*) [8] has occurred. As the system is driven harder and harder, the doubling occurs again and again. Eventually, the system response appears to be random noise. Such a transition to chaos has been observed in a common (albeit slightly unusual) system, a rectangular pot of boiling water [1].

The quasi-periodic route is less common. Although subharmonics do appear, they do not follow a period-doubling sequence. Rudnick [9] notes that initially

signals at irrational multiples of the driving frequency appear in the response. Phase locking [10], however, forces the system to respond at frequencies  $1/3$ ,  $1/5$ ,  $1/6$ , ...,  $1/n$  of the fundamental driving frequency (hence the origin of the term quasi-periodicity). It's interesting to note that the subharmonics usually occur in pairs. Moreover, the two subharmonics and the driving frequency make up a *resonant triad*—the sum of the two subharmonic wave numbers equals the wave number of the fundamental.

Most of the experimental work done to date has demonstrated the transition to chaos in discrete systems [2,3]. Experiments with simple continuous systems (such as a fluid) are relatively rare. However, at least one computer simulation of such a system has been carried out by treating the continuous model as a large number of discrete parts. The Navier-Stokes equations, for example, have been transformed into five coupled differential equations and the simulated fluid modeled on a computer [11]. The simulated system exhibited period-doubling. One of the classical resonant acoustical systems is a column of air in a closed pipe. Because of its simplicity, we chose to examine this system for period-doubling behavior.

### 1.3 Possible Acoustic Experiments

Many types of acoustical systems may exhibit a transition to chaos. We shall examine some of the possibilities and mention some of the more notable experiments that have been performed. Some of these experiments are discussed in detail in Part 2 (below). The discussion is split into three parts: discrete, hybrid, and continuous systems.

#### Discrete systems

Many discrete mechanical systems exhibit a transition to chaos. All of these systems have elements which are nonlinear and most take the period-doubling route to chaos. Some acoustical systems may be modeled with nonlinear, discrete (i.e., *lumped*) elements. Thus, one might expect to find chaos in any acoustical system which has an analogous, chaotic mechanical counterpart. Two systems come to mind: (1) a piston in a shallow, fluid-filled cavity [12] and (2) a Helmholtz resonator [13]. Both are analogous to a mass on a nonlinear spring. At present, no one has reported chaotic behavior for these acoustical systems. However, since the mechanical counterpart of these two acoustical systems *has* been studied extensively, finding chaos in these acoustical systems, although possible, is a less attractive prospect than the closed resonance tube.

#### Hybrid systems

A few acoustical "hybrid" experiments have been performed with great success [14,15,16]. These experiments are characterized by the introduction of an artificial



nonlinearity into an otherwise continuous linear (or nearly so) system. The transition to chaos in all of these experiments has been via period-doubling. We discuss them at length in Part 2.

### **Continuous systems**

Reports of observation of chaos in continuous systems are rare, both for mechanical and acoustical systems. Perhaps the first place one might look for chaos is in a simple resonant system (e.g., a cavity or standing wave tube). Indeed, various experiments have been performed using both systems.

Subharmonic generation in resonant cavities [17–20] has been studied extensively. Here the purpose of the work was only to examine the subharmonics, not to look for chaos. These systems may, in fact, exhibit chaotic behavior via a quasi-periodic transition.

Many experiments with standing, finite amplitude plane waves in tubes have been performed [21–27]. The researchers were generally *not* looking for subharmonics or chaos. However, careful study of these experiments has shown that a standing wave tube exhibits behavior characteristic of a hard spring/mass system. When driven hard enough, lumped element hard spring/mass systems have exhibited period-doubling.

## **2 HYBRID AND CONTINUOUS SYSTEMS**

### **2.1 Acoustic systems with an artificial nonlinearity**

Although systems with an artificial nonlinearity were an aside to our project, three papers deserve special mention:

1. Pedersen [14] did an early experiment which showed period-doubling in a strongly driven loudspeaker.
2. Maganza et al. [15] at IRCAM did an experiment showing period-doubling bifurcations in clarinet-like systems. A nonlinearity (a clarinet reed) was introduced into a simple linear system (a tube of air). The period-doubling bifurcations were quite distinct. The authors were also able to show that the system made a complete transition to chaos.
3. Kitano et al. [16] used a column of air, an amplifier, loudspeaker, and microphone for their experiment. The nonlinearity they introduced into the system was electrical—a simple full-wave rectifier. Two rather interesting things to which we return to later ought to be noted: (1) the transition to chaos was via period-doubling and (2) the onset of chaos was very abrupt and accompanied by a very distinct change in the sound character of the loudspeaker. Experiments similar to this one have recently been reported by two Chinese groups [28].

## **2.2 Continuous acoustical systems**

What follows is a discussion of various experiments with resonant cavities (acoustic interferometers) and some experiments with acoustic turbulence. Two groups seem to have done most of the interferometer work, one at University of Tennessee and the other at Harvard. Acoustic turbulence has been studied by Lauterborn's group at Göttingen.

### **Fabry-Perot interferometer**

The first observation of subharmonic generation in a resonant cavity seems to have been by Korpel and Adler [29]. A water-filled Fabry-Perot interferometer was parametrically excited at one of its higher order modes. Subharmonic responses were observed at frequencies  $f/n$  (where  $n$  is an integer). Several papers from University of Tennessee followed shortly thereafter [17-20]. The results of these experiments seem to indicate the authors had observed a partial quasi-periodic transition to chaos. A similar experiment with nearly the same results was performed a bit later at the Naval Postgraduate School (NPS) by Ruff [30].

### **Coupled oscillators**

The work Breazeale began was taken up by Yen at Harvard [31,32]. Yen developed a theory to describe the results obtained by the Tennessee group and expanded on them with some of his own experimental work. His report begins with a discussion of a discrete case: three coupled, weakly nonlinear oscillators. The discussion continues with an introduction to phase locking and "resonant triads." The analysis for the discrete case is then extended to the continuous case (finite elements). Using this method, Yen had quite a bit of success in predicting the behavior of the interferometer. His experiment [32] supports his theory quite well.

### **Acoustic turbulence**

The experiments done by Lauterborn's group [33-35] with a cavitating liquid are of a completely different character. In fact, Lauterborn prefers to call his work a study of "acoustic turbulence." Numerous bifurcations and eventually chaos were observed in the acoustical signal; the bubbles themselves contribute the nonlinearity to the system [36]. Here again the ubiquitous period-doubling route to chaos was observed.

## **3 CONTINUOUS SYSTEMS (Finite-Amplitude Standing Waves)**

Our goal was to try to observe subharmonics associated with a period-doubling transition to chaos in a resonance tube. Driving the tube at 2, 4, 8, or even 16 times the fundamental frequency would allow the appropriate subharmonics to

appear as lower order modes of the tube. To explore the possibility, we examined previous studies of finite-amplitude waves in a resonance tube. Perhaps hints of subharmonic behavior (or lack thereof) could be found.

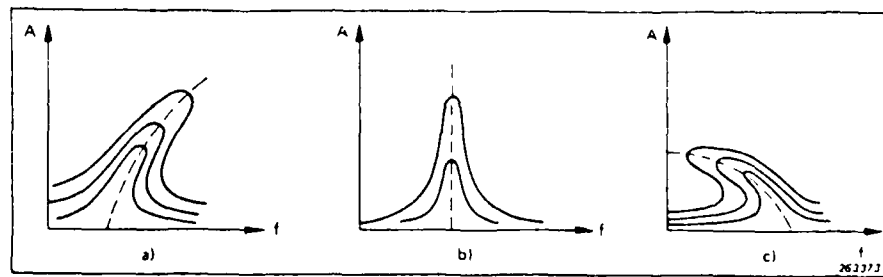
### **3.1 Sudden appearances (and disappearances) of shocks**

Studies of standing waves of finite amplitude in tubes go back to the early 1930s (see [21,22]). Early researchers were puzzled by the sudden appearance (and disappearance) of shocks in the tube at a certain drive amplitude. A similar effect was observed by varying the frequency at a fixed drive amplitude. As the frequency of the source approached the natural resonance frequency of the tube, shocks would suddenly appear at some point; as the source frequency was increased beyond resonance, the shocks disappeared as suddenly as they had appeared. Such curious behavior puzzled researchers for a long time. Reichwein [24] in 1962, in fact, published an entire thesis on just that behavior. These on/off appearances are suggestive of bifurcations and are also vaguely reminiscent of the onset of chaos heard by Kitano et al. [16].

### **3.2 Asymmetric response curves**

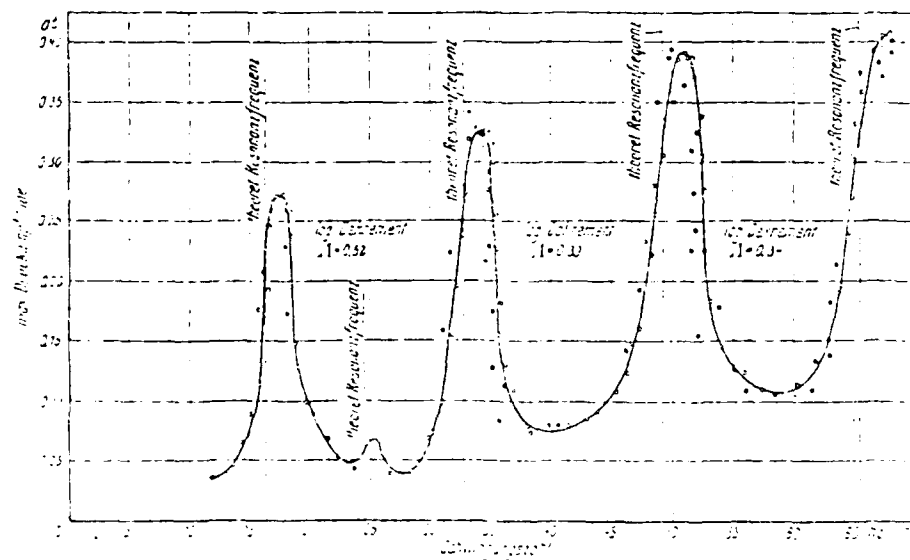
One way to understand the behavior of a system is to examine its frequency response (amplitude versus frequency). For a linear system, the frequency response curve is symmetric about the maximum, which occurs at the system's natural resonance frequency. However, for a mass on a nonlinear spring, a curious asymmetry develops. The natural frequency of the system depends on amplitude [37], and the response curve becomes skewed. Figure A-1 [38] shows the effect quite clearly. If the system is driven hard enough the response curve becomes multivalued and the system unstable. Interestingly, a certain amount of asymmetry may be observed in the response curves of the resonance tubes used in past high intensity experiments. What indication do these curves give? What follows is a list of what was found.

1. Mayer-Schuchard [21] had peak pressure amplitudes of 0.41 atm. His response curves, shown in Fig. A-2, are noticeably asymmetric and reminiscent of a hard spring/mass system. However, the curves do not appear to be skewed enough for the system to be unstable.
2. Lettau [22] had pressures of 0.2 to 0.3 atm and did not notice any asymmetry in his response curves.
3. Reichwein [24] had pressures of 0.28 atm. Although he did not find an asymmetric response curve, he did have problems finding the resonance frequency at high source levels. It was about 1 Hz higher than he had expected. He attributed the problem to a rise in temperature of the tube during the course of the experiment. In fact, later researchers [39] using the same apparatus



Typical resonance curves for various levels of excitation for:  
a) A hardening spring type resonant system  
b) A linear resonant system  
c) A softening spring type resonant system

Fig. A-1. Response curves for nonlinear and linear systems [38].

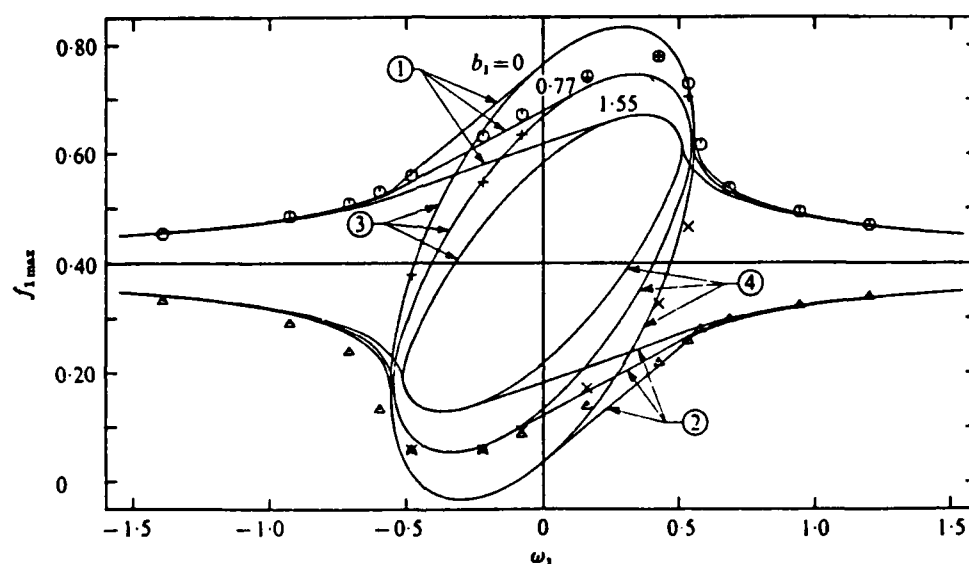


Aufg. 35 (lin.) 1st. Resonanz curve der Rohr 14

Fig. A-2. Mayer-Schuchard's response curves for standing waves in a resonance tube [21].

went to great lengths to correct the "problem." It did not go away. In hindsight, the change in resonance frequency was probably due to the nonlinear response of the system, something they had probably neglected.

4. Cruikshank [25] also observed an asymmetric response curve. He attributed the asymmetry to nonlinear behavior and made the analogy with the hard spring/mass system.
5. The work of Sturtevant [26] is by far the most interesting of these studies. Sturtevant obtained **very** intense levels with peak pressures on the order of 0.82 atm. His response curve is shown in Fig. A-3. The horizontal axis shows



Response curve for closed tube. —, nonlinear theory: ①, maximum value of  $f_1$ ; ②, minimum value of  $f_1$ ; ③,  $f_1$  just behind discontinuity; ④,  $f_1$  just ahead of discontinuity. Experiment: ○, maximum  $f_1$ ; △, minimum  $f_1$ ; +,  $f_1$  behind shock; ×,  $f_1$  ahead of shock. File 94-19 (cf. table 2).

Fig. A-3. Sturtevant's response curves [26].

departures from the natural resonance frequency of the system (at  $\omega_1 = 0$ ); the vertical axis the maximum amplitude ( $f_{1max}$ ). The open circles are the data we are interested in. They clearly exhibit a noticeable asymmetry. However, even at the levels Sturtevant was able to maintain, he notes a difference in the expected and the observed resonance of only about 1%. The lack of a substantial shift in the resonance peak probably means that it would be very difficult to achieve a multivalued response curve and the concomitant jump phenomena that signal a bifurcation. Without a bifurcation, the way seems blocked to substantial subharmonic response (and subsequent chaos).

### 3.3 Experiments to examine subharmonics

Only one experiment to date has been done to look specifically for subharmonics. That work was done at Naval Postgraduate School by Donnelly [27]. His experiment is in fact what we had in mind. He excited his tube at twice the lowest resonance frequency and looked for subharmonics *at the lowest resonance*. No subharmonics were observed. However, his peak levels were only about 160 dB (peak pressures less than 0.1 atm). He did not attempt to drive his tube at higher amplitudes—the seal around his piston began leaking and the  $Q$  of his system dropped dramatically. In general,  $Q$  is lowered by increases in both tube losses and nonlinear effects. Cruikshank encountered similar problems—one of his figures showing the loss of  $Q$  as the level in the tube is increased is reproduced in Fig. A-4. One can only

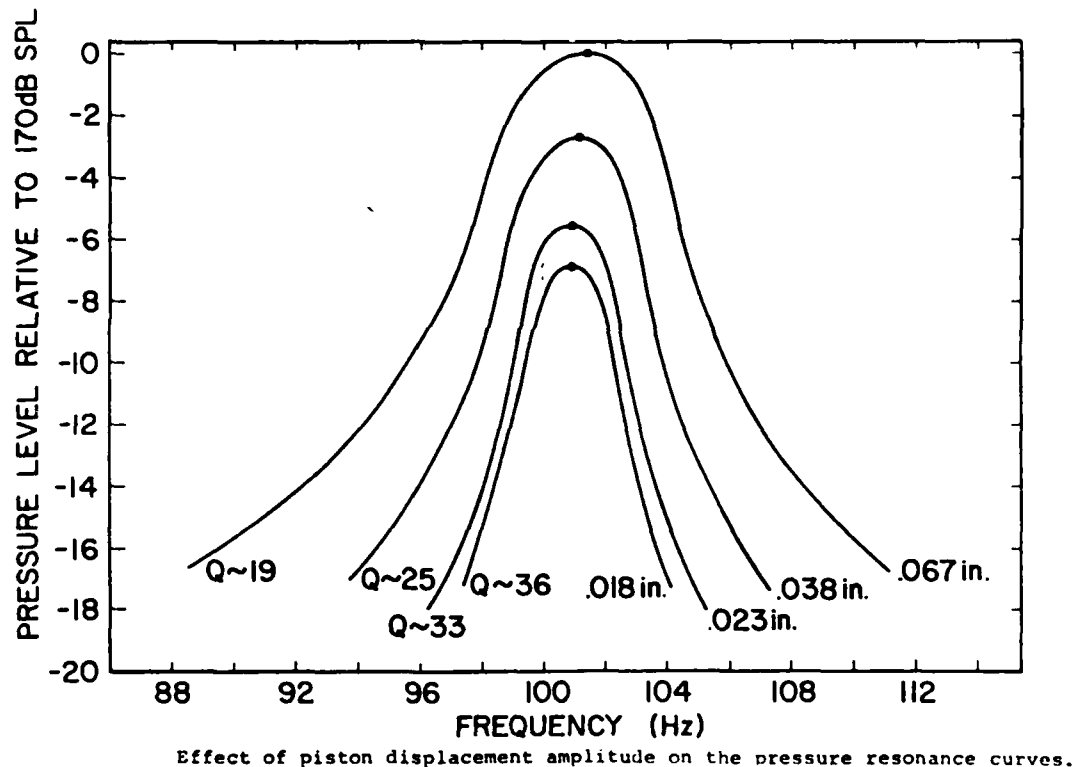


Fig. A-4. Cruikshank's response curves showing loss of  $Q$  with drive amplitude [25].

conclude that pressures must be greater than 0.1 atm, and Sturtevant's results show that little subharmonic response can be expected even up to 0.82 atm.

#### 4 CONCLUSION

It seems that our proposed experiment has not proved fruitful. Even if we could get up to the levels Sturtevant obtained,\* we would not be assured of seeing subharmonic response. Moreover, the resulting increase in level would also result in a dramatic lowering of the system's  $Q$  and broadening of the response curve. A broad response curve is much less likely to become multivalued than a narrow one. Thus, we must conclude that a standing wave tube is not a good place to search for chaos.

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\*This would take considerable expense of labor and money: Sturtevant's piston was connected to a 15 hp motor and the piston to a water-cooled pipe. The whole system was mounted on a 1 ton bed isolated from the laboratory floor by air springs.

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Predecessor Contract N00014-75-C-0867

Code	ONR CONTRACTS
B = chapter in a book	0574 means N00014-84-K-0574, began 8-1-84
J = journal publication	0867 means N00014-75-C-0867, ended 8-31-84
JS = submitted for journal publication	0805 means N00014-82-K-0805, ended 10-31-85
O = oral presentation	
P = paper in a proceedings	
T = thesis or dissertation	
TR = technical report	

1984

ONR Contract	Code	
0867	O,P	1. S. Saito and T. G. Muir, "Simple model for analyzing a parametric focusing source," Meeting of the Acoustical Society of Japan, 29-31 March 1984, Tokyo, Japan. PAPER: Proceedings, pp. 33-40 (in Japanese).
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ONR  
Contract    Code

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| 0867         | J | *5. M. F. Hamilton and F. H. Fenlon, "Parametric acoustic array formation in dispersive fluids," J. Acoust. Soc. Am. <u>76</u> , 1474-1492 (1984).  |
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\* Primary support for this work came from ONR Contract N00014-79-C-0624 with Pennsylvania State University.

\*\* Primary support for this work came from NASA Grant NSG 3198.

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| 0867                 | J    | 6.    | D. T. Blackstock, "Generalized Burgers equation for plane waves," J. Acoust. Soc. Am. <u>77</u> , 2050-2053 (1985).   |
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\*\*\* A portion of Hamilton's support for this work came from Contract N00014-85-K-0708.

1986

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